Notes:

- Read Course Information: Section 6 (Miscellaneous) and Section 8 (Academic Dishonesty or Misconduct).
- When you are giving a construction, example, etc., provide a justification with your argument. Your solutions to numerical problems must contain the derivation of your answers. In all of your presentations, strive for correctness, completeness, and clarity. When in doubt about the assumptions of problems, the interpretations of wording, etc., consult the instructor.
- You should strive to complete all problems assigned, and a subset of them will be graded.

1. Read the notes above carefully.
3. A metric $d$ on a set $X$ is a function $d : X \times X \to \mathbb{R}_{\geq 0}$ (where $\mathbb{R}_{\geq 0}$ denotes the set of all nonnegative reals) that satisfies, for all $x, y, z \in X$: (1) [nonnegativity] $d(x, y) \geq 0$ and equality holds when and only when $x = y$, (2) [symmetry] $d(x, y) = d(y, x)$, and (3) [subadditivity / triangle inequality] $d(x, y) \leq d(x, z) + d(z, y)$.

The following function $D$ is employed in the definition of “relative definition in a true subtraction”. Define the function $D : (\mathbb{R} - \{0\}) \times (\mathbb{R} - \{0\}) \to \mathbb{R}_{\geq 0}$ by:

$$D(x, y) = \frac{|x - y|}{\max\{|x|, |y|\}}$$

for all $x, y \in \mathbb{R}_{\geq 0}$.

Does $D$ satisfy the triangle inequality? Prove your answer.

4. Do [Cha10] Chapter 2, Section 2.6, exercises 1 and 2.
5. Do [Cha10] Chapter 3, Section 3.1, exercise 1 (a), (b), (j), and (k).
7. In [Cha10] Chapter 3, section 3.2: One “small problem”, it is stated that the function $f : \mathbb{R} \to \mathbb{R}$ defined by:

$$f(x) = \begin{cases} \exp\left(-\frac{1}{x^2}\right) & \text{if } x \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

has the “mother of all flat spots” at $x = 0$. Prove that for all $n \geq 0$, $f^{(n)}(0) = 0$.

8. More problem(s) will be given in later version.