

CS 3613: Theoretical Foundations of Computing Spring 2009

Homework 1: Preliminaries

Due Date: January 26 (Monday), 2009

Notes:

- Read Course Information: Section 7 (Miscellaneous) and Section 9 (Academic Dishonesty or Misconduct).
- When you are giving a construction, example, etc., provide a justification with your argument. Your solutions to numerical problems must contain the derivation of your answers. In all of your presentations, strive for correctness, completeness, and clarity. When in doubt about the assumptions of problems, the interpretations of wording, etc., consult the instructor.
- You should strive to complete all problems assigned, and a subset of them will be graded.

1. Read the notes above carefully.
2. Do [Sip06] Chapter 0, exercises 0.3, 0.6, and 0.7. Read [Sip06] pages 10 – 13 on Graphs, and do [Sip06] Chapter 0, exercise 0.8.
3. In each case below, find an expression for the indicated set that involves A , B , and C , and the three set-theoretic operators \cup , \cap , and \neg (complementation):

- (a) $\{x \mid x \in A \text{ or } x \in B \text{ but not both}\}$.
- (b) $\{x \mid x \text{ is an element of exactly one of the three sets } A, B, \text{ and } C\}$.
- (c) $\{x \mid x \text{ is an element of exactly two of the three sets } A, B, \text{ and } C\}$.
- (d) $\{x \mid x \in A \text{ or } x \in B \text{ or } x \in C\}$.

4. Read/review “binary relation”, “equivalence relation”, “equivalence class”, and “index of an equivalence relation” in a typical discrete mathematics text, and do the following problem.

Let P denote the set of all compound propositions involving the simple/atomic propositions p , q , and r and the logical connectives \vee , \wedge , and \neg (complementation). (Included in P are the tautology proposition *true* and the contradiction proposition *false*.) Define a binary relation R on P by:

$$s R t \text{ if and only if } s \equiv t,$$

where \equiv denotes the logical equivalence in propositional logic.

- (a) Show that R is an equivalence relation on P .
 - (b) How many equivalence classes of R are there? [For every element $p \in P$, the equivalence class (of the equivalence relation R on P) containing p , denoted by $[p]_R$, is the set $\{t \in P \mid t R p\}$ — the set of all elements in P that are related to p under R . The index of an equivalence relation is the number of its equivalence classes.] List some elements in the equivalence class containing the compound proposition $(p \wedge q) \vee (\neg r)$. List some elements in the equivalence class containing the tautology *true*, and some elements in the equivalence class containing the contradiction *false*.
5. Write a quantified statement that says there are exactly two elements x in the set A for which the proposition $P(x)$ holds.
 6. An alphabet is a non-empty finite set of symbols, and a string over the alphabet is a finite sequence of symbols of the alphabet. Some example strings over the binary alphabet $\{0, 1\}$ are: 1011 (for the sequence $(1, 0, 1, 1)$), 10 (for the sequence $(1, 0)$), ϵ (denoting the empty sequence).

For strings x and y over an alphabet, we denote by $|x|$ the length of the sequence x , and by xy the concatenation of the two sequences x and y in that order.

For each integer $n \geq 0$, we define the strings x_n and y_n over the alphabet $\{0, 1\}$ as follows: $x_0 = 0$ and $y_0 = 1$, and for $n \geq 1$, $x_n = x_{n-1}y_{n-1}$ and $y_n = y_{n-1}x_{n-1}$. Prove the following statements using mathematical induction:

- (a) For every $n \geq 0$, $|x_n| = |y_n|$.
- (b) For every $n \geq 0$, x_n and y_n differ in every position.
- (c) For every $n \geq 0$, x_{2n} and y_{2n} are palindromes. (A string x is a palindrome if the reversal sequence of x is identical to the sequence x .)
- (d) For every $n \geq 0$, x_n contains neither the substring 000 nor the substring 111. (A string x is a substring of a string y if x is simply a contiguous subsequence of y .)