

1. The examination contains 4 problems. You have 90 minutes for 40 points.
 2. Show all important steps in your work. Your answers will be graded on its correctness and clarity.
-

1. [10 points]

- (a) [6 points] Let $L = \{a^i b^j c^k \mid i, j, k \geq 0, i + k = j\}$. Prove that L is not regular by using the pumping lemma for regular languages.

- (b) [4 points] Using the closure properties of regular languages (regularity-preserving operators) to show that L is not regular.

2. [10 points] Give context-free grammars generating the following languages.

Notes: No conversion from pushdown automaton to context-free grammar is accepted. For each grammar, give a brief and precise interpretation of each variable (that is, the set of strings derived from the variable).

(a) [5 points] L_1 , the set of all non-palindromes over alphabet $\{a, b\}$. (A palindrome over an alphabet Σ is a string over Σ that reads the same backward and forward; what is a non-palindrome?)

- (b) [5 points] $L_2 = \{a^m b^n c^p d^q \mid m, n, p, q \geq 0, m + n = p + q\}$. Note that the condition $m + n = p + q$ does not necessarily imply that $m = q$ and/or $n = p$.

3. [10 points] Consider the following language:

$$L = \{z \in \{a, b\}^* \mid z \text{ is of the form } xyx \text{ for some } x \text{ and } y \text{ with } |x| \geq 1\}.$$

For examples: The strings aaa (with $x = a$ and $y = a$) and $abababa$ (with $x = a$ and $y = babab$, or $x = aba$ and $y = b$) are in L , but the string $aaab$ is not in L .

Decide if L is regular. Prove your answer.

4. [10 points] Give the 6-tuple definition of a pushdown automaton for the language:

$$L = \{a, b\}^* - \{ww \mid w \in \{a, b\}^*\}.$$

Note: A direct construction of a pushdown automaton is required. Give a brief and precise interpretation of the states and transitions of your machine.