Notes:

- When you are giving a construction, example, etc., provide a justification with your argument. Your solutions to numerical problems must contain the derivation of your answers. In all of your presentations, strive for correctness, completeness, and clarity. When in doubt about the assumptions of problems, the interpretations of wording, etc., consult the instructor.
- You should strive to complete all problems assigned, and a subset of them will be graded.

1. Read the notes above carefully.

2. Provide regular expressions for following languages over the alphabet \( \Sigma = \{0, 1\} \). Annotate your regular expressions or provide brief explanations for your answers.
   (a) The language of all strings with at most one pair of consecutive 1s.
   (b) The language of all strings not containing 010 as a substring.
   (c) The language of all strings that end in even number of 0s.
   (d) The language of all strings with exactly one occurrence of the substring 000.

3. Recall that for two regular expressions (over an alphabet) \( r \) and \( s \), \( r = s \) means that \( L(r) = L(s) \). Prove or disprove the following for regular expressions \( r \) and \( s \) over an alphabet \( \Sigma \).
   (a) \((rs + r)^* r = r(sr + r)^*\)
   (b) \((r + s)^* = r^* + s^*\)
   (c) \((r^* s^*)^* = (r + s)^*\)

4. For each of the following languages, prove its non-regularity by a direct application of Pumping Lemma:
   (a) \( L_1 = \{0^i1^j0^j \mid i, j \geq 0\}\).
   (b) \( L_2 = \{0^i \mid i \geq 0\}\).

5. [ Not to be included in Homework 3. ] Consider the following languages over the alphabet \( \Sigma = \{a, b\}\):
   (a) \( L_1 = \{xyx^r \mid x, y \in \Sigma^+\}\).
   (b) \( L_2 = \{xx^r y \mid x, y \in \Sigma^+\}\).
   For each of the above languages, prove or disprove its regularity.

6. (a) [ Not to be included in Homework 3. ] Using the closure properties of regular languages and that the language \( \{a^n b^n \mid n \geq 0\} \) is not regular, prove that the language \( L = \{w \in \{a, b\}^* \mid \#_a(w) \neq \#_b(w)\}\) is not regular.
   (b) [ Not to be included in Homework 3. ] Note: Bonus will be given for correct oral presentation of the following problem to the instructor by the due date; written solution will not be graded/accepted.
   Prove that the above language \( L \) is non-regular by a direct application of Pumping Lemma.