Notes:

- Read Course Information: Section 7 (Miscellaneous) and Section 9 (Academic Dishonesty or Misconduct).
- When you are giving a construction, example, etc., provide a justification with your argument. Your solutions to numerical problems must contain the derivation of your answers. In all of your presentations, strive for correctness, completeness, and clarity. When in doubt about the assumptions of problems, the interpretations of wording, etc., consult the instructor.
- You should strive to complete all problems assigned, and a subset of them will be graded.

1. Read the notes above carefully.

2. Let \( \Sigma \) be an alphabet. Clearly there exist two distinct strings \( x \) and \( y \) over \( \Sigma \) that satisfy the commutativity: \( xy =yx \) (for instance, when \( x = \epsilon \)).

   Is the commutativity possible for non-empty strings \( x \) and \( y \) (in \( \Sigma^+ \))? Prove that this can not happen, or describe/prove precisely the circumstances under which it can.

3. Let \( \Sigma \) be the alphabet \( \{0,1\} \). Denote by \( L \) the language \( \{ u \in \Sigma^* \mid u = vv \text{ for some string } v \in \Sigma^* \} \). Prove or disprove that the language \( L \) can be expressed as the concatenation of two “non-trivial” languages \( L_1 \) and \( L_2 \) over \( \Sigma \): \( L_1 \neq \{ \epsilon \} \) and \( L_2 \neq \{ \epsilon \} \) and \( L = L_1L_2 \) (where \( \epsilon \) denotes the empty string).