Notes:

• Read Course Information: Section 7 (Miscellaneous) and Section 9 (Academic Dishonesty or Misconduct).

• When you are giving a construction, example, etc., provide a justification with your argument. Your solutions to numerical problems must contain the derivation of your answers. In all of your presentations, strive for correctness, completeness, and clarity. When in doubt about the assumptions of problems, the interpretations of wording, etc., consult the instructor.

• You should strive to complete all problems assigned, and a subset of them will be graded.

1. Read the notes above carefully.

2. Give the 5-tuple definition and state-transition diagram for a finite automaton (deterministic, nondeterministic, or nondeterministic finite automaton with Λ-transitions) accepting each of the following languages. Briefly and precisely give the interpretations of the states and transitions in your construction.
   
   (a) The set of all strings in \( \{0, 1\}^* \) such that, at some place in the string, there are two 0s separated by an even number of symbols.

   (b) The set of all strings in \( \{a, b, c\}^* \) such that exactly one of the two symbols \( a \) or \( b \) appears at least three times in all.

   (c) The set \( \text{half}(L) \) of all first halves of strings in a regular language \( L \) over an alphabet \( \Sigma \), that is,

   \[
   \text{half}(L) = \{ x \in \Sigma^* \mid \exists y \ (xy \in L \text{ and } |x| = |y|) \}.
   \]

3. Convert the following nondeterministic finite automaton with \( \epsilon \)-transitions, \( M \), to an equivalent nondeterministic finite automaton \( M_1 \), and then using the Subset Construction to convert \( M_1 \) to an equivalent deterministic finite automaton \( M_2 \) with its inaccessible states removed. Explicitly and briefly write down each step which you perform, such as: (1) Computing all the \( \epsilon \)-closures of the states of \( M \), and showing complete state-transition diagrams of \( M_1 \) and \( M_2 \).

\[\begin{align*}
1 & \quad \text{start} \\
2 & \quad a \\
3 & \quad b, \epsilon \\
4 & \quad b \\
\end{align*}\]

\[\begin{align*}
1 & \quad \epsilon \\
3 & \quad \epsilon \\
\end{align*}\]

Figure 1: A nondeterministic finite automaton with \( \epsilon \)-transitions.

4. For two strings \( u \) and \( v \) over an alphabet, \( u \) is obtained from \( v \) by discarding symbols if it is possible to discard zero or more symbols from \( v \) to result in the string \( u \) – denoted by \( u = \text{disc}(v) \).
For example, the following strings can all be obtained from 0110 by discarding symbols:

$\epsilon, 0, 1, 00, 01, 10, 010, 011, 110, \text{and } 0110$.

Let $\Sigma = \{0, 1\}$, and assume that $L \subseteq \Sigma^*$ is a regular language.

(a) Prove the regularity of the language:

$$J = \{ x \in \Sigma^* \mid \exists y \in L \ (x = \text{disc}(y)) \}.$$  

(b) Prove the regularity of the language:

$$K = \{ x \in \Sigma^* \mid \exists y \in L \ (y = \text{disc}(x)) \}.$$  

5. Do [Sip12] Chapter 1, exercise 1.18 (h) and (i) (in exercise 1.6). Justify your solutions by properly structuring and annotating your regular expressions.

6. More problems will be given in later version(s).